CH2 STANDARD MODEL

2.1 Gauge symmetries

2.1.1 Abelian case : QED

→ Free field $\mathcal{L}$ of QED has $U(1)_g$ symmetry. Making it local,
$$\mathcal{L} = \bar{\Psi}_e (i\not{D} - m_e)\Psi_e \text{ with } D_\mu \equiv \partial_\mu + ie A_\mu$$
The $U(1)_L$ symmetry allows the vector field to propagate:
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$ is gauge invariant, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

PROP The QED $\mathcal{L}$ is build as $\{\mathcal{L}_{free}$ + localization principle + vector field kinetic term $\}$

↳ $\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}_e (i\not{D} - m_e)\Psi_e$

↳ It gives an interaction term $\bar{\Psi}_e\, e\, Q_e \gamma_\mu A^\mu \Psi_e \equiv e A^\mu J_\mu^{e-m}$
conserved : $\partial_\mu J^\mu_{em} = 0 \Rightarrow Q_{em} = \int J_0^{em} d^3x = Q_e(\#(e^-) - \#(e^+))$

2.1.2 Non abelian gauge symmetry:

→ The free $\mathcal{L}$ of 2 fermions of the same mass displays a $SU(2)_g$ → doublet.
$$\mathcal{L} = \sum_{i=1,2} \bar{\Psi}_i (i\not{\partial} - m)\Psi_i = \bar{\Psi}(i\not{\partial} - m)\Psi \text{ with } \Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \text{ such that}$$
$$\Psi \mapsto \Psi' = U\Psi,\ U = \exp\{-i \tfrac{\tau_a}{2}\theta_a\},\ \tau_a = \sigma^a$$
↳ Localization principle:
$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi$ with $D_\mu - ig \frac{\tau_a}{2} A_{\mu a}$ ($\# A_\mu = \#$ generator of the group)
with $\Psi \mapsto U\Psi$ so that
$$\frac{\tau}{2}\cdot A'_\mu = U \frac{\tau}{2}\cdot A_\mu U^{-1} - \frac{i}{g}\partial_\mu U . U^{-1}$$

$$\Leftrightarrow A'^a_\mu = A^a_\mu + \underbrace{C_{abc}\,\theta_b A^c_\mu}_{\text{global}} - \frac{1}{g}\underbrace{\partial_\mu \theta_a}_{\text{local}} \quad , \ C_{abc} \equiv \text{group structure constant.}$$

PROP $\{A^a_\mu\}$ is a triplet of $SU(2)_g$: it transforms in the adjoint rep of $SU(2)$

DEF The Field Strength tensor $F^a_{\mu\nu}$ is defined as $-ig\frac{\tau}{2}\cdot F_{\mu\nu} = [D_\mu, D_\nu]$
$\Leftrightarrow F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\varepsilon_{abc} A^a_\mu A^b_\nu$

2.2 E-W interaction for leptons and quarks

→ Electroweak interactions $\Leftrightarrow$ localization of $SU(2)_L$ and $U(1)_Y$,
L-fermions $\in$ doublet, R-fermions $\in$ singlets. We have:

$$L_e = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix},\ L_\mu = \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix} \text{ and } L_\tau = \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix} \text{ with } Y_{L_e} = Y_{L_\mu} = Y_{L_\tau} = -1$$

$$e_R,\quad \mu_R \quad \text{and } \tau_R \qquad \text{with } Y_{e_R} = Y_{\mu_R} = Y_{\tau_R} = -1$$

$$Q_L^u = \begin{pmatrix} u_L \\ d_L \end{pmatrix},\ Q_L^c = \begin{pmatrix} c_L \\ s_L \end{pmatrix} \text{ and } Q_L^t = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad Y_{Q_L^u} = Y_{Q_L^c} = Y_{Q_L^t} = -1/3$$

$$u_R,\quad c_R \quad \text{and} \quad t_R \quad \text{with} \quad Y_{u_R} = Y_{c_R} = Y_{t_R} = 4/3$$

$$d_R,\quad s_R \quad \text{and} \quad b_R \quad \text{with} \quad Y_{d_R} = Y_{s_R} = Y_{b_R} = -2/3$$

→ The E-W $\mathcal{L}$ is given by

$$\mathcal{L}_{SM}^{EW} = \sum_{\ell = e,\mu,\tau} \left(i \bar{L}_\ell \not{D} L_\ell + i \bar{\ell}_{R\ell} \not{D} \ell_{R\ell} \right)$$

$$+ \sum_{q = u,c,t} \left(i \bar{Q}_L^q \not{D} Q_L^q + i \bar{q}_R^q \not{D} q_R^q \right)$$

$$+ \sum_{\bar{q} = d,s,b} \left(i \bar{q}_R^{\bar{q}} \not{D} q_R^{\bar{q}} \right) - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

with $D_\mu = \partial_\mu - i g_2 \frac{\tau}{2} \cdot W_\mu - i g_1 \frac{Y}{2} B_\mu$

→ We define $\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & s\theta_W \\ -s\theta_W & c\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$ and $\sin\theta_W \equiv \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$,
one gets

$$\mathcal{L} \ni e J_\mu^{em} A^\mu + \frac{g_1}{\cos\theta_W} J_\mu^Z Z^\mu \text{ with } J_\mu^{em} = \sum_F Q_F \bar{\Psi}_F \gamma_\mu \Psi_F$$

$F \in \{\text{fermions}\}$

$Q_F = T_{3F} + Y_F/2$

$\hookrightarrow e = g_1 \sin\theta_W$

$\hookrightarrow J_\mu^Z = \sum_{F = F_L, F_R} \{ T_{3F} - Q_F \sin^2\theta_W \} \bar{\Psi}_F \gamma_\mu \Psi_F$

2.3 QCD interactions for quarks

→ Strong interaction = gauge principle with quark = triplet of SU(3).

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{k=u,d,s,c,b,t} \bar{q}_k (i\gamma^\mu D_\mu - m_k) q_k$$

with $q_k \equiv \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix} = q_{kL} + q_{kR}$ and $G^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g_s \cdot f_{abc} A^b_\mu A^c_\nu$

where $[T_a, T_b] = i f_{abc} T_c$, and $T_a = \frac{\lambda_a}{2}$ the 8 gellman matrices

$D_\mu = \partial_\mu - ig \frac{\lambda_a}{2} A_{\mu a}$

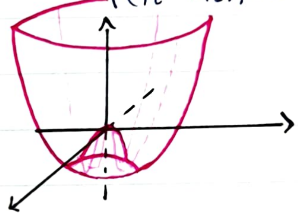
2.4 Spontaneous Symmetry breaking of $SU(2)_L \times U(1)_Y$

2.4.1 SSB of a $SU(2)_L$:

→ Consider $\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu a} F^{\mu\nu}{}_a$ with $\phi = \begin{pmatrix} (\phi_1 + i\phi_2)/\sqrt{2} \\ (\phi_i - i\phi_2)/\sqrt{2} \end{pmatrix}$

and $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$; $\mu^2 < 0$

↳ The vacuum is a S^3: $\phi_1^2 + \dots + \phi_4^2 = \frac{-\mu^2}{\lambda} = v^2$



We consider the vacuum to be $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$, $v \in \mathbb{R}$.

In polar coord.:

$$\phi(x) = \underbrace{\exp\left\{ i \tau_a \frac{\xi_a(x)}{v} \right\}}_{\text{tangential fields}} \underbrace{\begin{pmatrix} 0 \\ \frac{v + \eta(x)}{\sqrt{2}} \end{pmatrix}}_{\text{radial field}} \quad (\phi_1 = \phi_2 = \phi_4 = 0\ ;\ \phi_3 = (v + \eta)\ )$$

→ Gauge choice: $\phi \mapsto \phi' = U\phi = \exp\{-i \tau_a \cdot \xi_a / v\} . \phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$

The $\mathcal{L}$ becomes:

$$\mathcal{L} = (D_\mu \phi')^\dagger (D^\mu \phi') - \frac{\mu^2}{2}(\eta + v)^2 - \frac{\lambda}{4}(\eta + v)^4 - \frac{1}{4} F'^a_{\mu\nu} F'^{\mu\nu}_a$$

$$= \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{g_1^2}{8} \left(0 \quad \frac{\eta + v}{\sqrt{2}}\right) (\tau_a \cdot W'_{a\mu}) (\tau_b \cdot W'_{b\mu}) \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$$

$$- \frac{\eta^2}{2}(\mu^2 + 3\lambda v^2) - \eta(\mu^2 v + \lambda v^3) - \lambda v \eta^3 - \frac{1}{4}\eta^4 - \frac{1}{4} F'^2$$

$$= \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{1}{4}\left(\frac{gv}{2}\right)^2 (W'^a_\mu W'^{a\mu}) - \frac{\eta^2}{2}(-2\mu^2) - \lambda v \eta^3$$

$$- \frac{1}{4}\lambda \eta^4 + \frac{g_1^2 v^2}{8}(2\eta v + \eta^2) W'^a_\mu W^\mu_a - \frac{1}{4} F'^2$$

↳ ① η BEH boson with $m_\eta^2 = -2\mu^2 = v^2/2\lambda$ ② massive W_μ^a: $m^2_{W_a} = \left(\frac{gv}{2}\right)^2$

③ no ξ^a left.

2.4.2 SSB of $SU(2)_L \times U(1)_Y$:

→ Assume $Y_\phi = 1 \Rightarrow Q_{\phi_3} = 0 \Leftrightarrow$ No SSB of $U(1)_{em}$

→ $$\mathcal{L} \ni (D_\mu \phi')^\dagger (D^\mu \phi) \ni \frac{1}{2}(0 \; v)\left(\frac{g_1}{2}\tau \cdot W_\mu + \frac{g_2}{2} B_\mu\right)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$
$$= \frac{v^2}{8}\left\{ g_1^2\left((W_\mu'^1)^2 + (W_\mu'^2)^2 \right) + (-g_1 W_\mu'^3 + g_2 B_\mu)^2 \right\}$$
$$= \underbrace{\frac{g_1^2 v^2}{4}}_{m_W^2} W_\mu'^{+} W'^{\mu -} + \underbrace{\frac{(g_1^2+g_2^2)v^2}{8}}_{m_Z^2} \underbrace{\left(\frac{g_1}{\sqrt{g_1^2+g_2^2}} W_\mu'^3 - \frac{g_2}{\sqrt{g_1^2+g_2^2}} B_\mu\right)^2}_{Z^\mu Z_\mu}$$

↳ 3 massive gauge boson $W^\pm, Z$, 1 massless A_μ

↳ prediction $m_W^2 / m_Z^2 = \cos\theta_W = g_1^2/(g_1^2+g_2^2)$

→ $$\mathcal{L}_{EW} = -\frac{1}{4} W_{a\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{F = F_L, F_R} \bar{F}\, i\partial\!\!\!/ F$$
$$+ m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu$$
$$+ \frac{g_1}{\sqrt{2}}\left(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}\right) + e J_\mu^{em} A^\mu$$
$$+ \frac{g_1}{\cos\theta_W} J_\mu^Z Z^\mu$$
$$- \frac{m_\eta^2}{2}\eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$
$$+ \frac{g_1^2}{4}(2\eta v + \eta^2)(W_\mu^+ W^{-\mu})$$
$$+ \frac{g_1^2+g_2^2}{8}(2\eta v + \eta^2) Z_\mu Z^\mu$$

$J_\mu^Z = \sum g_{F_L} \bar{F}_L \gamma_\mu F_L + g_{F_R} \bar{F}_R \gamma_\mu F_R$

$g_{F_L} = T_3^F - Q_F \sin^2\theta_W$

$g_{F_R} = -Q_F \sin^2\theta_W$

↳ Only 1 scalar doublet and 4 parameters: g_1, g_2, μ^2, λ

9

2.5 Fermion masses

→ Since $Y_\phi = 1$, an extra term, the Yukawa interaction is allowed:

$\mathcal{L} \ni y_e \bar{L}_e \phi e_R + h.c.$

$= -\frac{y_e v}{\sqrt{2}} \bar{e}_L e_R - \frac{y_e h}{\sqrt{2}} \bar{e}_L e_R + h.c.$

→ 3 generations of leptons and quarks:

$\mathcal{L} \ni -\bar{L}_i \, y_{ij} \, \phi \, e_{Rj} + h.c.$

$\ni (M_e)_{ij} \bar{e}_{iL} e_{jR} - y^L_{ij} \frac{1}{\sqrt{2}} \bar{e}_{iL} e_{jR} + h.c.$ with $(M_e)_{ij} = -\frac{v}{\sqrt{2}} y^L_{ij}$, and

lepton mass

$\mathcal{L} \ni -\bar{Q}_{Li} \phi y^D_{ij} d_{Li} d_{Rj} - y^D_{ij} \frac{1}{\sqrt{2}} d_{Li} d_{Rj} = \underbrace{-\frac{v}{\sqrt{2}} y^D_{ij}}_{(M_D)_{ij}} d_{Li} d_{Rj} - y^D_{ij} \frac{1}{\sqrt{2}} d_{Li} d_{Rj}$

→ Since $\tilde{\phi} \equiv i \tau_2 \phi^*$ transforms as ϕ under $SU(2)_L$, we can write

$\mathcal{L} \ni -\bar{Q}_{Li} \tilde{\phi} y^U_{ij} u_{Rj} + h.c.$

$\ni \underbrace{-\frac{v}{\sqrt{2}} y^U_{ij}}_{(M_U)_{ij}} \bar{u}_{Li} u_{Rj} - y^U_{ij} \frac{1}{\sqrt{2}} \bar{u}_{Li} u_{Rj} + h.c.$

$(M_U)_{ij}$ quark mass

→ To diagonalize M_e, M_D, M_U, we perform a bi-unitary transfo.

$\mathcal{L} \ni -\bar{e}'_L \underbrace{U^\dagger_{eL} M_e U_{eR}}_{M_e^{Diag}} e'_R - \bar{d}'_L \underbrace{U^\dagger_{dL} M_D U_{dR}}_{M_D^{diag}} d'_R - \bar{u}'_L \underbrace{U^\dagger_{uL} M_U U_{uR}}_{M_U^{diag}} u'_R$

→ No change for leptons: $(\bar{\nu}_L \; \bar{e}_L) \, i \not{D} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (\bar{\nu}'_L \; \bar{e}'_L) \, i \not{D} \begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix}$

→ Big ≠ for the quarks: $U_{dL} \neq U_{uL}$

→ Neutral currents: remain diag no FCNC in SM.

→ Charged currents: not flavour diag. in physical basis.

$\mathcal{L} \ni \frac{g}{\sqrt{2}} \left[\bar{u}'_L \gamma_\mu (U^\dagger_{uL} U_{dL}) d'_L \right] W^{+\mu} + h.c.$

$= \frac{g}{\sqrt{2}} (\bar{u}'_L, \bar{c}'_L, \bar{t}'_L) \gamma_\mu U^\dagger_{uL} U_{dL} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}$

V_{CKM} ⇝ 3 angles, 1 phase: δ_{CKM}

→ δ_{CKM} break CP (unique in SM).

→ Leptons: $L = L_e + L_\mu + L_\tau$ conserved, as well as each $\underset{U(1)_{L_e}}{L_e}$, $\underset{U(1)_{L_\mu}}{L_\mu}$ and $\underset{U(1)_{L_\tau}}{L_\tau}$.

→ Quarks: only $B \equiv (\# q - \# \bar{q})/3$ is conserved ⇝ $U(1)_B$

2.6 Some examples

→ The SM comes from experimental facts. For instance, QED is the most precisely measured theory in physics.

⊙ Delta-0 decay: $\Delta^0 = (u\,d\,d)$

→ We observe $\Delta^0 \to p + \pi^0$ with $\tau \sim 10^{-24}\,s$
and $\Lambda^0 \to p + \pi^0$ with $\tau \sim 10^{-10}\,s$

→ In term of decay rate $\Gamma = 1/\tau \propto \int d\pi\, |\mathcal{M}|^2 \propto m g^2$
$[\Gamma] = E$

One has $\frac{\Gamma_\Delta}{\Gamma_\Lambda} = 10^{14} \simeq \frac{g_\Delta^2}{g_\Lambda^2}$. for $g_\Delta = g_3 \sim 1$, $g_\Lambda \sim 10^{-7}$

↳ 3 types of interaction: QED, Weak and Strong

2.7 Limitations of the SM

⊙ Neutrinos mass:

→ In the SM, $m_\nu = 0$, but experiments $\Rightarrow m_{\nu_i} \sim 10^{-2}\,eV$.
If $m_\nu \Leftarrow$ SSB, we would have $y_{\nu_i} \sim 10^{-12}$

⊙ Baryon asymmetry:

→ $\eta \equiv n_B / n_\gamma \simeq 6.10^{-10}$
↳ Sakharov conditions: 1) B not conserved
2) violation of C, P and CP
3) out of equilibrium evolution.

→ Possibility: $U(1)_Y \times SU(2)_L \times SU(3) \subset G$ → $SU(5)$, → $SO(10)$

→ Chiral anomalies?